Horizontal Heat Conduction during Electron Precipitation into the Upper Atmosphere

J A Gledhill

Department of Physics
Rhodes University, Grahamstown

A simple model is set up to consider the effects of horizontal conduction of heat away from a region of electron precipitation in the upper atmosphere. It is shown that a given precipitated flux produces a much smaller increase of temperature in a region of the order of 1 km in width than it does in one several hundred km wide. A method is developed and illustrated, by which it is possible to determine whether heating of the neutral atmosphere is negligible during such precipitation events, thus allowing the use of static atmospheric models in the theoretical treatment of the effects.

'n Eenvoudige model word saamgestel om die effek van die horisontale geleiding van hitte weg van 'n gebied van elektron-presipitasie in die bo-atmosfeer, te beskou. Dit blyk dat 'n gegewe gepresipiteerde vloed 'n veel kleiner temperatuur-toename in 'n gebied van sowat 1 km breed as in 'n gebied van etlike honderde kilometer breed teweegbring. 'n Metode waar-volgens bepaal kan word of verhitting van die neutrale atmosfeer tydens sodanige presipitasie onbeduidend is, word ontwikkel en geïllustreer. Daardeur word die gebruik van statiese atmosferiese modelle by die teoretiese behandeling van die betrokke effekte moontlik gemaak.

Introduction

It is customary, when calculating the effects produced in the upper atmosphere by precipitated electrons, to assume a suitable static model of the neutral atmosphere and then to estimate the rate of energy deposition as a function of height, as if the neutral gas distribution was unaffected by the continuous influx of energy (e.g. *Rees*, 1963, 1964; *Walt*, 1967; *Stolarski*, 1967; *Berger et al.*, 1970; *Wulff & Gledhill*, 1973). On the other hand, it is widely believed that the atmosphere in the auroral zone is considerably modified by heating due at least in part to the precipitation of electrons there (*Chamberlain*, 1961; *Blamont & Lory*, 1964). There is a growing conviction that this heating may be the source of some travelling ionospheric disturbances and storm-time changes in the ionosphere at lower latitudes. Heating by precipitated electrons has been proposed as the reason for high electron temperatures observed by satellites (e.g. *Willmore*, 1964) at middle latitudes. Nevertheless, spectroscopic determinations of rotational temperatures (e.g. *McPherson & Vallance-Jones*, 1960) and of doppler line widths (e.g. *Hilliard & Shepherd*, 1966) show only small increases above the neutral temperatures to be expected in the absence of aurora, and have even been used to estimate upper atmospheric temperatures as if there was no local heating due to the precipitation of the charged particles (e.g. *Hunten*, 1961; *Brandy*, 1965). *Electron* temperatures in auroras, on the other hand, are known to be very high (e.g. *Walker & Rees*, 1968). It appears at first glance as if there is a contradiction between these various assumptions.

In this paper a simple model is set up and solved to obtain estimates of the order of magnitude of the temperature increases to be expected in various appropriate circumstances. It is shown that the rate of local temperature rise depends on the width of the region over which the particles are being precipitated, and that the absence of considerable increases in temperature, even in bright auroras, is not inconsistent with the occurrence of widespread heating and expansion of the atmosphere when comparable particle fluxes are precipitated over extended regions.

Horizontal Heat Conduction due to Localized Precipitation

We consider an infinite, horizontal "slab" of the atmosphere, thin enough for the temperature to be assumed uniform throughout before the onset of local heating. We take the z axis as pointing vertically upwards out of the slab and the x and y axes as lying in the horizontal plane. Initially the temperature is T_0 everywhere, but at time $t = 0$ heating commences suddenly, at a rate Q_0 per unit volume per unit time, between $x = a/2$ and $x = -a/2$, at all values of y. The heat conduction problem is thus a one-dimensional one in the x-direction and the equation of continuity for energy may be written as

$$Q(x, t) = \rho C_v \frac{\partial T(x, t)}{\partial t} - \kappa \frac{\partial^2 T(x, t)}{\partial x^2} \qquad (1)$$

where ρ is the density, C_v the specific heat capacity at constant volume and κ the thermal conductivity, all of which are assumed to be constant. We may write the heat production rate as

$$Q(x, t) = Q_0 \Pi \left(\frac{x}{a}\right) H(t) \qquad (2)$$

where $\Pi\left(\frac{x}{a}\right)$ is the unit rectangular pulse of width a and H(t) is the Heaviside unit step function.

If we write the temperature in the form

$$T(x, t) = T_0 + \Theta(x, t) \qquad (3)$$

equation (1) becomes

$$Q_0 \, \Pi \left(\frac{x}{a}\right) H(t) = \rho \, C_V \frac{\partial \, \Theta(x, t)}{\partial t} - \kappa \frac{\partial^2 \Theta(x, t)}{\partial x^2} \qquad (4)$$

This equation may be solved by successive Laplace and Fourier transformations; the solutions have also been given by *Carslaw & Jaeger* (1947). If we introduce the new variables

$$X = \frac{x}{a} \qquad (5)$$

and

$$Y = \frac{4\kappa}{\rho C_V} \frac{t}{a^2} = 4\,K \frac{t}{a^2} \qquad (6)$$

where

$$K = \frac{\kappa}{\rho C_V} \qquad (7)$$

is the diffusivity, the solutions are

$$\Theta(x, t) = \frac{Q_0 \, a^2}{4\kappa} \Big[-(2X^2 + \tfrac{1}{2}) + \left\{ \frac{Y}{2} + |X - \tfrac{1}{2}|^2 \right\} \operatorname{erf}\left(\frac{|X - \frac{1}{2}|}{\sqrt{Y}}\right) + \left\{ \frac{Y}{2} + (X + \tfrac{1}{2})^2 \right\} \operatorname{erf}\left(\frac{X + \frac{1}{2}}{\sqrt{Y}}\right) + \sqrt{\frac{Y}{\pi}} \, |X - \tfrac{1}{2}| \exp\left(-\frac{|X - \frac{1}{2}|^2}{Y}\right) + \sqrt{\frac{Y}{\pi}} (X + \tfrac{1}{2}) \exp\left(-\frac{(X + \frac{1}{2})^2}{Y}\right) \Big]$$

$$= \frac{Q_0 \, a^2}{4\kappa} D_1(X, Y); \quad (\tfrac{1}{2} \geqslant X \geqslant 0) \qquad (8)$$

and

$$\Theta(x, t) = \frac{Q_0 \, a^2}{4\kappa} \Big[-2X - \left\{ \frac{Y}{2} + (X - \tfrac{1}{2})^2 \right\} \operatorname{erf}\left(\frac{X - \frac{1}{2}}{\sqrt{Y}}\right) - \left\{ \frac{Y}{2} + (X + \tfrac{1}{2})^2 \right\} \operatorname{erf}\left(\frac{X + \frac{1}{2}}{\sqrt{Y}}\right) - \sqrt{\frac{Y}{\pi}} (X - \tfrac{1}{2}) \exp\left(-\frac{(X - \frac{1}{2})^2}{Y}\right) + \sqrt{\frac{Y}{\pi}} (X + \tfrac{1}{2}) \exp\left(-\frac{(X + \frac{1}{2})^2}{Y}\right) \Big]$$

$$= \frac{Q_0 \, a^2}{4\kappa} D_2(X, Y); \quad (X \geqslant \tfrac{1}{2}) \qquad (9)$$

together with two symmetrical expressions for negative values of X.

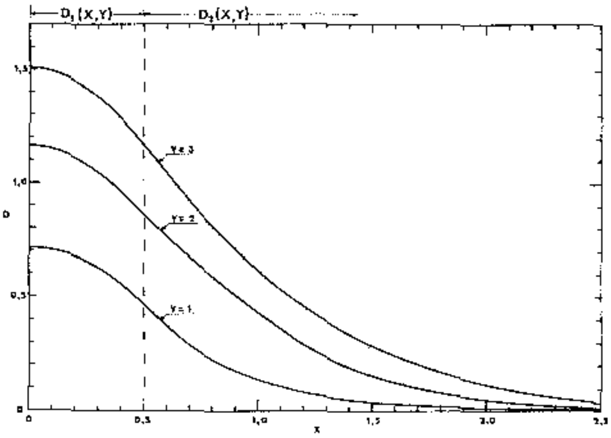

Fig. 1. The functions $D_1(X, Y)$ and $D_2(X, Y)$.

Some examples of the functions $D_1(X, Y)$ and $D_2(X, Y)$ are shown in Fig. 1. As we would expect, the maximum temperature occurs in the centre of the heating region, at $X = 0$, and the maximum temperature gradient, and therefore the maximum heat flux also, at the edges of the heating zone, where $X = \pm \frac{1}{2}$. Although the expressions (8) and (9) appear complicated, we may note that the heat production rate Q_0 occurs only in the multiplying factor on the right hand side of each equation, so that we may immediately draw the conclusion that:

(*i*) *the temperature increase* Θ (x, t) *is directly proportional to the rate of heat input* Q_0 *at all times and places.*

This is, of course, a consequence of the linearity of the heat continuity equation.

We shall be interested primarily in the maximum temperature increase, at $X = 0$. From equations (5) and (8) we find

$$\Theta(0, t) = \frac{Q_0 \, a^2}{4\kappa} D_1(0, Y)$$

$$= \frac{Q_0 \, a^2}{4\kappa} \left[(Y + \tfrac{1}{2}) \operatorname{erf}\left(\frac{1}{2\sqrt{Y}}\right) + \sqrt{\frac{Y}{\pi}} \exp\left(-\frac{1}{4Y}\right) - \tfrac{1}{2} \right] \qquad (10)$$

The function $D_1(0, Y)$ is shown in Figure 2. The initial slope at $Y = 0$ is readily shown to be unity, so that we may write

$$D_1(0, Y) \doteqdot Y; \quad (Y << 1) \qquad (11)$$

Thus the temperature is given by

$$\Theta(0, t) \doteqdot \frac{Q_0}{\rho C_V} t; \quad (t/a^2 << 1/4K) \qquad (12)$$

which is the value it would have at all times if no heat was removed by conduction.

Equation (10) is too complex for its significance in any particular case to be easily seen. It is shown in the appendix, however, that it approaches the simple form

$$D_1(0, Y) \doteqdot 1{,}129\sqrt{Y} - 0{,}5; \quad (Y >> 1) \qquad (13)$$

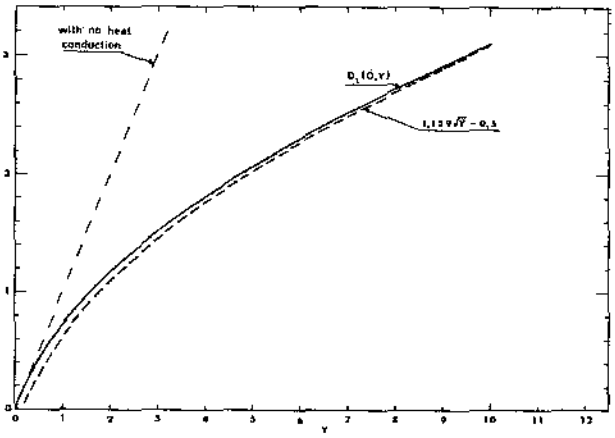

Fig. 2. The functions $D_1(0, Y)$ and $1{,}129\sqrt{Y} - 0{,}5$.

This function is also shown in Fig. 2. It is seen to be a remarkably good approximation even when Y does not exceed unity by very much. The error is in fact less than 10% for $Y > 1{,}3$ and less than 1% for $Y > 9$. Clearly, as Y increases, the constant term ($-0{,}5$) eventually must become negligible, although it only becomes less than 1% of the first term when $Y > 2\,000$. From equations (6), (8) and (13) we then have

$$\Theta(0,t) \doteqdot \frac{0{,}565}{\sqrt{\rho C_V \kappa}} Q_0\, a\sqrt{t} - \frac{Q_0\, a^2}{8\kappa};\ (t/a^2 >> 1/4K) \quad (14)$$

and from the preceding discussion

$$D_1(0,\ Y) \doteqdot 1{,}129\sqrt{Y};\quad (Y > 2\,000) \quad (15)$$

and

$$\Theta(0,t) \doteqdot \frac{0{,}565}{\sqrt{\rho C_V \kappa}} Q_0\, a\sqrt{t};\ (t/a^2 > 500/K) \quad (16)$$

By comparing equations (12) and (16) we can draw two further conclusions:

(*ii*) *For a given* a, *when* t *is small the temperature increase is proportional to* t, *but as time goes on the removal of heat by conduction reduces the dependence, which eventually becomes one of proportionality to* $\sqrt{t}$ *after a long time;*

(*iii*) *after a given time, if* a *is small the temperature increase is proportional to* a, *but as* a *is increased the dependence becomes less marked, until for large values of* a, Θ *is independent of it.*

The latter conclusion clearly offers a possible way of explaining the small temperature increases observed in auroral features, compared with the considerable thermal expansion which undoubtedly takes place in the much wider auroral zones. The efficacy of this explanation can only be tested by substituting appropriate numerical values in the formulae.

Numerical Calculations

The quantity $1/4K = \rho C_V/4\kappa$ is obviously of some importance in discussing the heating of the upper atmosphere by electron bombardment. The heat capacity per unit volume can be found from kinetic theory

$$\rho C_V = \sum_i n_i C_i k$$

where n_i is the number density of the molecular species i, C_i is 3/2 for a monatomic gas and 5/2 for a diatomic one and k is Boltzmann's constant. The thermal conductivity κ is given by

$$\kappa = 1{,}125 \times 10^{14} \sqrt{T}\ \ \text{eV cm}^{-1}\,\text{s}^{-1}\,\text{K}^{-1} \quad (18)$$

(deduced from the values given by *Herman & Chandra* (1969)).

In order to gain some feeling for the orders of magnitude involved, we shall choose arbitrarily the CIRA (1965) model 3 atmosphere for midnight, which has an exospheric temperature of 816 K. From the tables of this model we find the values of 1/4K shown in Table 1. To illustrate the discussion further it will be convenient to consider the value of Y after a suitable time t: we shall choose t = 900 s (= 15 minutes). Table 2 gives values of Y after this time for four decades of a, calculated from the figures in Table 1 and equation (6). The ranges in which the temperature increases may be approximated by equations (11) and (12), (13) and (14), and (15) and (16) are evident from the table. From the appropriate equations we now derive Table 3, showing the temperature increases Θ (0,900) in terms of the heat input rate Q_0.

The attainment of independence of $\Theta(0{,}900)$ on a, as discussed under conclusion (iii) above, when a is large, is well illustrated in the column of the 140 km level.

Table 1

Values of $1/4K = \rho C_V/4\kappa$ for CIRA model 3 atmosphere, 0h

z	140	200	300	(km)
$\rho C_V/4\kappa$	$1{,}8 \times 10^{-9}$	$8{,}7 \times 10^{-11}$	$4{,}1 \times 10^{-12}$	(s cm^{-2})

Table 2

Values of Y after 900 s

z		140	200	300	(km)
a	1	5	10^4	2×10^4	
(km)	10	5×10^{-2}	10^2	2×10^2	
	100	5×10^{-4}	1	2	
	1000	5×10^{-6}	10^{-2}	2×10^{-2}	

To estimate actual temperature increases, we need to know the heat input rate Q_0. This is related to the rate at which energy is deposited by the precipitated electrons. This has been calculated by several workers, e.g. *Rees* (1963, 1964); *Berger et al.* (1970); *Wulff & Gledhill* (1973). The energy deposited per unit volume per unit time is proportional to the incident flux, J_0, but its distribution in height depends in a complex way on the energy spectrum and pitch angle distribution of the electrons, the atmospheric model and the magnetic dip angle. If we are content to confine our attention to orders of magnitude only, however, we may note from Fig. 3 of *Wulff & Gledhill* (1973) that the production rates of ion pairs by unit incident flux of electrons which are isotropic in pitch angle over the downward hemisphere and have energies in the auroral range, 1 - 10 keV, are of the order of 10^{-5} cm^{-1} at 140 km, 10^{-6} cm^{-1} at 200 km and 10^{-7} cm^{-1} at 300 km.

Each ion pair produced involves the loss of an average of 35 eV by the ionizing particle. Some of this energy is conducted away by the ambient electron gas, some is radiated and some remains as local thermal energy. *Roble & Dickinson* (1973) have quoted figures which lead to an estimate that about 30% of this total energy remains as local thermal energy of the neutral gas. Using these figures, we can rewrite Table 3 in terms of the incident omnidirectional integral electron flux as in Table 4.

We note the sensitivity of the region around 200 km, where a flux of only 10^8 electrons $cm^{-2}s^{-1}$ would produce a temperature rise of 8 K in 900 s in the centre of a wide band of precipitation. The same flux would only produce a temperature increase of 0,1 K in a feature 1 km wide. There seems to be little doubt that lateral conduction of heat plays an important role in cooling narrow, bright auroral features.

Typical auroral electron fluxes have been considered by *Rees* (1969). From his figures we can estimate that a bright aurora is produced by an omnidirectional integral flux of the order of 10^{10} $cm^{-2}s^{-1}$. In a narrow, bright feature, Table 4 then leads us to expect temperature increases of the order of 10 K after 15 minutes at the 200 km level. If the same flux was to precipitate over an area 100 km wide, however, temperature increases of the order of 700 K might be expected in the same time, with consequent radical redistribution of the neutral atmosphere. We may note from Table 2, that $Y << 1$ under these conditions, so that equation (11) applies and $\Theta(0, t)$ is proportional to t. Thus we may expect the temperature to rise at roughly 1 K per second and our static model of the atmosphere will become inapplicable after a few seconds.

Discussion

There are two main results of the present study which should be stressed. The first is the demonstration that lateral heat conduction in the ionosphere is efficient enough to limit the temperature rise in narrow regions of electron precipitation, so that increases of temperature in a region of the order of 1 km in width may be only a few degrees, compared with increases of several hundred degrees if the same flux were to be precipitated over an area several hundred kilometres in width. The relationship is by no means a simple one, but estimates can be made in particular cases by the method outlined. Numerical values seem to be in accordance with experimental observations.

The second result is the establishment of a method by which a criterion can be found for the flux above which atmospheric heating becomes so severe that it can no longer be neglected in the theoretical treatment of the problem. Here, again, the relationships involved are not simple, but is is possible to make suitable estimates quickly by using the approximate method of *Wulff & Gledhill* (1973) to relate the heat input rate at a given height to the electron flux, in conjunction with the equations set out in the present paper for the rate of temperature rise. If it should appear that the temperature is increased by a significant amount by the energy input from the precipitated electrons, the problem of determining the vertical profile of energy depostion becomes a difficult one in atmospheric dynamics. This problem has not, to the author's knowledge, been discussed in the literature.

It should be noted that the simple model used in the present paper takes account only of heat conduction in

Table 3

Temperature increase after 900 s in terms of Q_0

	z	140	200	300	(km)
a	1	$7{,}6 \times 10^{-6}\ Q_0$	$9{,}3 \times 10^{-5}\ Q_0$	$1{,}3 \times 10^{-4}\ Q_0$	
(km)	10	$4{,}8 \times 10^{-6}\ Q_0$	$8{,}9 \times 10^{-4}\ Q_0$	$1{,}3 \times 10^{-3}\ Q_0$	
	100	$4{,}8 \times 10^{-6}\ Q_0$	$7{,}5 \times 10^{-3}\ Q_0$	$9{,}0 \times 10^{-3}\ Q_0$	
	1000	$4{,}8 \times 10^{-6}\ Q_0$	$8{,}2 \times 10^{-3}\ Q_0$	$1{,}6 \times 10^{-2}\ Q_0$	

Table 4

Orders of magnitude of Θ (0,900) in terms of J_0

	z	140	200	300	(km)
a	1	$8 \times 10^{-10}\ J_0$	$9 \times 10^{-10}\ J_0$	$1 \times 10^{-10}\ J_0$	
(km)	10	$5 \times 10^{-10}\ J_0$	$90 \times 10^{-10}\ J_0$	$10 \times 10^{-10}\ J_0$	
	100	$5 \times 10^{-10}\ J_0$	$700 \times 10^{-10}\ J_0$	$90 \times 10^{-10}\ J_0$	
	1000	$5 \times 10^{-10}\ J_0$	$800 \times 10^{-10}\ J_0$	$200 \times 10^{-10}\ J_0$	

the horizontal plane. Some heat will be conducted away in the vertical direction, adding to the heat flux down the temperature gradient which is normally present in the atmosphere at all times. By virtue of the linearity of the equation of continuity, the problem could be solved separately for the heat input from the precipitated electrons and the resulting temperatures added to those previously existing. Unfortunately, the distribution of the heat input rate Q in height depends on the energy and pitch angle spectra of the precipitating particles and the temperature profile can only be determined by numerical integration of the heat input profile in each case. Since the purpose of the present paper is not to calculate the temperature increase under such conditions, but merely to establish criteria for its neglect, the matter will not be considered further here. It is worth noting, however, that the energy removed by vertical conduction will reduce the temperature increases calculated for the present simple model, and so will tend to make the criteria more conservative than would otherwise have been the case. The more general problem is receiving further attention.

Acknowledgements

The author records his gratitude to the South African Council for Scientific and Industrial Research for a Senior Research Bursary, to the Director of the Radio and Space Research Station of the British Science Research Council at Slough, where the work was carried out, and to his colleagues at the Radio and Space Research Station, especially Dr H Rishbeth, for useful discussions and criticism.

References

Berger, MJ, Seltzer, SM & Maeda, K Energy deposition by auroral electrons in the atmosphere. *J. atmosph. terr. Phys.*, **32**, 1015–1045, 1970.

Blamont, JE & Lory, ML Sudden heating of the ionosphere in the auroral zone. *Nature, Lond.*, **201**, 593–595, 1964.

Brandy, JH A temperature curve for the upper atmosphere from 80 to 160 km based on 3914 Å N_2^+ rotational temperatures in aurora. *Can. J. Phys.*, **43**, 1697–1700, 1965.

Carslaw, HS & Jaeger, JC *Conduction of heat in solids.* Oxford, Clarendon Press, 1947.

Chamberlain, JW *Physics of the aurora and airglow*, 297–299. New York & London, Academic Press, 1961.

CIRA (1965) COSPAR *International Reference Atmosphere* 1965. Amsterdam, North Holland Publishing Co., 1965.

Herman, JR & Chandra, S The influence of varying solar flux on ionospheric temperatures and densities: a theoretical study. *Planet. Space Sci.*, **17**, 815–840, 1969.

Hilliard, RL & Shepherd, CG Upper atmospheric temperatures from doppler line width - IV. *Planet. Space Sci.*, **14**, 383–406, 1966.

Hunten, DM Temperatures deduced from aurora and airglow spectra. *Annls. Geophys.*, **17**, 249–255, 1961.

McPherson, DH & Vallance-Jones, A A study of the latitude dependence of OH rotational temperatures for Canadian stations. *J. atmosph. terr. Phys.*, **17** 302–308, 1960.

Rees, MH Auroral ionization and excitation by incident energetic electrons. *Planet. Space Sci.*, **11**, 1209–1218, 1963.

Rees, MH Note on the penetration of energetic electrons into the earth's atmosphere. *Planet. Space Sci.*, **12**, 722–725, 1964.

Rees, MH Auroral electrons. *Space Science Reviews*, **10**, 413–441, 1969.

Roble, RG & Dickinson, RE Is there enough solar extreme ultra-violet radiation to maintain the global mean thermospheric temperature? *J. geophys. Res.*, **78**, 249–257, 1973.

Stolarski, RS Calculation of auroral emission rates and heating effects. *Planet. Space Sci.*, **16**, 1265–1276, 1967.

Walker, JCG & Rees, MH Ionospheric electron densities and temperatures in aurora. *Planet. Space Sci.*, **16**, 459–475, 1968.

Walt, M Penetration of electrons into the atmosphere. In *Aurora and Airglow*, edited by BM McCormac. New York, Reinhold, 1967.

Willmore, AP Ionospheric heating in the F-region. *Proc. R. Soc. A*, **281**, 140–149, 1964.

Wulff, A & Gledhill, JA Atmospheric ionization by precipitated electrons. *J. atmosph. terr. Phys.*, 1973 (in press).

Appendix

From equation (10), the function $D_1(0, Y)$ is given by

$$D_1(0,Y) = (Y+\tfrac{1}{2})\,\mathrm{erf}\left(\frac{1}{2\sqrt{Y}}\right) + \sqrt{\frac{Y}{\pi}}\exp\left(-\frac{1}{4Y}\right) - \tfrac{1}{2}$$

$$= (Y+2)\frac{2}{\sqrt{\pi}}\int_0^{\frac{1}{2\sqrt{Y}}} \exp(-u^2)\,du + \sqrt{\frac{Y}{\pi}}\exp\left(-\frac{1}{4Y}\right) - \tfrac{1}{2}$$

$$= (Y+\tfrac{1}{2})\frac{2}{\sqrt{\pi}}\int_0^{\frac{1}{2\sqrt{Y}}} (1 - u^2 + \ldots.)\,du$$

$$+ \sqrt{\frac{Y}{\pi}}\left(1 - \frac{1}{4Y} + \ldots.\right) - \tfrac{1}{2}$$

$$= (Y+\tfrac{1}{2})\frac{2}{\sqrt{\pi}}\left(\frac{1}{2\sqrt{Y}} - \ldots.\right) + \sqrt{\frac{Y}{\pi}}\left(1 - \frac{1}{4Y} + \ldots.\right) - \tfrac{1}{2}$$

$$= (Y+\tfrac{1}{2})\frac{2}{\sqrt{\pi}}\frac{1}{2\sqrt{Y}} + \sqrt{\frac{Y}{\pi}} - \tfrac{1}{2} + \left(\text{terms in } \frac{1}{Y}\right)$$

$$= \sqrt{\frac{4Y}{\pi}} - \tfrac{1}{2} + (\text{terms tending to zero as } Y \to \infty)$$

Thus

$$D_1(0, Y) \doteqdot 1{,}129\sqrt{Y} - 0{,}5; \quad (Y >> 1)$$

which is equation (13).

(Received 26 June 1973)